Three-dimensional Weight Function Analyses and Stress Intensity Factors for General Surface and Corner Crack Emaninating from Circular Hole

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1. Introduction

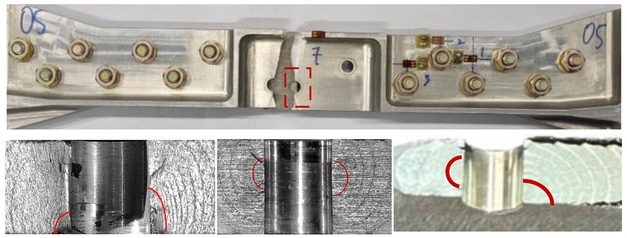

Fig.1 Complex hole-edge crack in airframe structure

◆Due to the **complexity of the problem, limited work** has been done for the SIFs of eccentric and asymmetric surface and corner hole-edge cracks, and combination of surface and corner hole-edge cracks subjected to complex loads.

◆**Accurate and highly efficient** methods for computation of the SIFs of such complex cracks in aircraft structure are highly demanded for damage tolerance analysis.

◆The **slice synthesis weight function method** is developed to meet this demand.

2. Complex corner and surface cracks at a circular hole

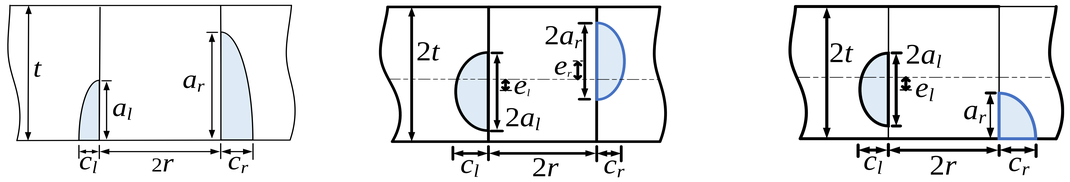

Fig.2 (a) Asymmetric corner cracks; (b) Asymmetric and eccentric surface corner cracks; (c) Combination of surface and corner cracks.

3. Slice Synthesis weight function method (SSWFM)

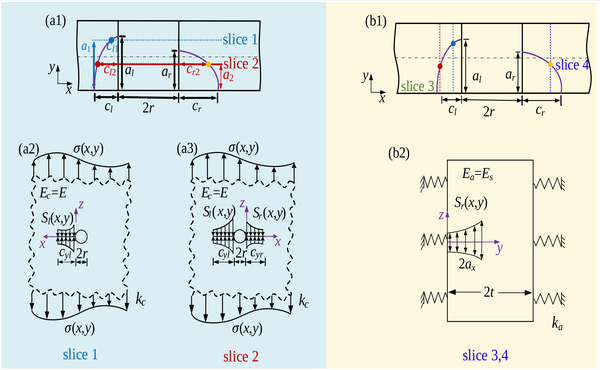

Fig.3 Decomposition of 3D crack based on SSWFM

◆**The relation between the 2D SIFs of the slice and 3D SIF $K(\varphi)$ is**

$$K_j(\varphi)=\frac{1}{1-\eta^2}\left\{K_{cj}^4(c_{yl},c_{yr})+\left[\frac{E_j}{E_{sj}}K_{aj}(a_{xj})\right]^4\right\}^{1/4}(-1)^n$$

K_c, K_a are SIFs of the slices

◆**The 2D SIFs of the slices are determined by the 2D weight function method**

$$K_{cj}(c_{yl},c_{yr})=\int_0^{c_{yl}}\left[\sigma_l(x,y)-S_l(x,y)\right]\cdot m_{cj}(c_{yl},c_{yr},r,x)\mathrm{d}x+\int_0^{c_{yr}}\left[\sigma_r(x,y)-S_r(x,y)\right]\cdot m_{cj}(c_{yl},c_{yr},r,x)\mathrm{d}x$$

$$K_{aj}(a_{xj})=\int_0^{a_{xj}}S_j(x,y)\,m_a(a_{xj}/t,y/t)\mathrm{d}y$$

$\sigma(x,y)$ is the uncrack stress from a stress analysis of the body without crack

$S(x,y)$ is the spring stress representing the interaction between the slices

◆**The spring stress $S(x, y)$ is determined by the displacement compatibility equations and 2D weight function method.**

$$\int_x^{c_{yl}}\int_0^{\xi}\left[\sigma_l(x,y)-S_l(x,y)\right]\cdot m_{cl}(\xi,c_{yr},r,x)\mathrm{d}x\cdot m_{cl}(\xi,c_{yr},r,x)\mathrm{d}\xi=\frac{E_l}{E_{sl}}\int_y^{a_{xl}}\int_0^{\xi}S_l(x,y)\cdot m_a(\xi/t,y/t)\mathrm{d}y\cdot m_a(\xi/t,y/t)\mathrm{d}\xi$$

$$\int_x^{c_{yr}}\int_0^{\xi}\left[\sigma_r(x,y)-S_r(x,y)\right]\cdot m_{cr}(c_{yl},\xi,r,x)\mathrm{d}x\cdot m_{cr}(c_{yl},\xi,r,x)\mathrm{d}\xi=\frac{E_r}{E_{sr}}\int_y^{a_{xr}}\int_0^{\xi}S_r(x,y)\cdot m_a(\xi/t,y/t)\mathrm{d}y\cdot m_a(\xi/t,y/t)\mathrm{d}\xi$$

4. 3D SIFs and verifications——hole edge cracks *in an infinite plate*

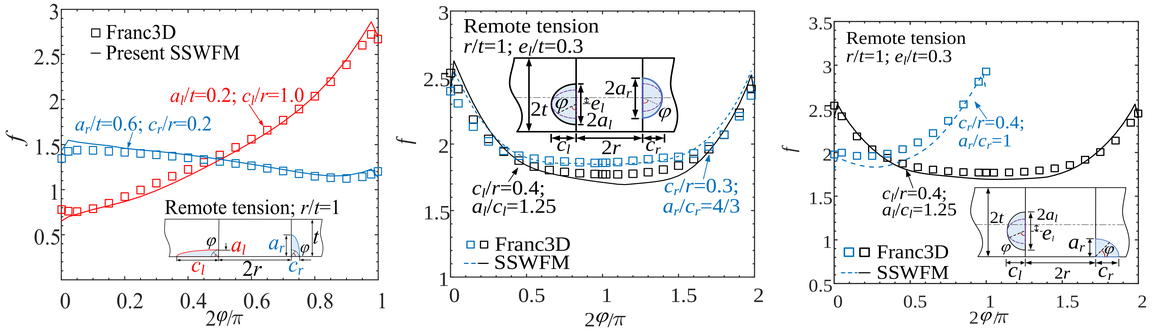

Fig.4 The SIFs from SSWFM agree well with those from FEM/Franc3D, most of the differences are less than 8%

5. 3D SIFs and verifications——hole edge cracks *in airframe structure*

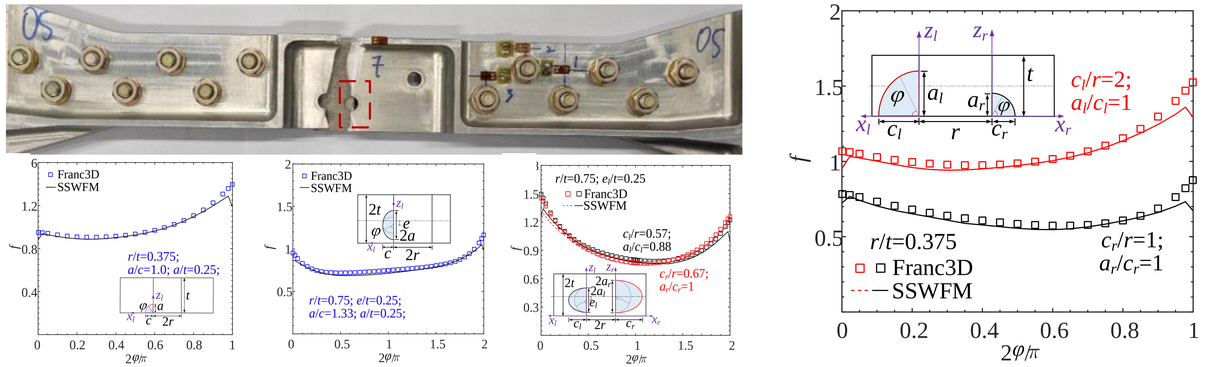

Fig.5 The SIFs from SSWFM agree well with those from FEM/Franc3D for 3D cracks in airframe, most of the differences are less than 10%.

6. Conclusions

① The SIFs obtained from the SSWFM agree well with those from FEM/Franc3D, ***most of the differences are less than 10%.***

② The present SSWFM is ***2-3 orders-of-magnitude faster*** than FEM/Franc3D in calculation of SIFs,

③ The SSWFM will significantly enhance fatigue crack growth analysis capability for ***engineering structures*** with circular hole.

References

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[4] Zhang B. Weight function analysis and application for typical three-dimensional hole-edge cracks in aircraft structure. PhD thesis, Shanghai Jiao Tong University, 2023.